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DENSITY AND TOTAL PRESSURE BEHAVIOR IN THE PROCESS OF FORCED AND SPONTANEOUS RECONNECTION

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The specific features of the density and total pressure (the sum of magnetic and gas-kinetic pressures) behavior in the process of forced (by the example of propagation of a magnetic acoustic wave in the vicinity of the X-point) and spontaneous (by the example of development of a helical Thirring mode) reconnection are considered. It is shown that the total pressure distribution depends weakly on the initial value of the gas-kinetic pressure, the strength of the z-component of the magnetic field, and the thermal conductivity in a wide range of parameters. The character of density distribution is determined only by the thermal conductivity. It is also shown that the behavior of the total pressure and density in the case of spontaneous reconnection weakly depends on the thermal conductivity, in contrast to the forced case.

1. We consider a two-dimensional ($\partial/\partial z = 0$) problem of propagation of a magnetic acoustic wave in the vicinity of the X-point. The initial equations of one-fluid magnetic hydrodynamics have the following form in commonly accepted dimensionless variables [1, 2]:

$$\frac{\partial A}{\partial t} + (\boldsymbol{V}\,\nabla)A = \nu\Delta A, \qquad \frac{\partial H_z}{\partial t} + (\boldsymbol{V}\,\nabla)H_z = -H_z\operatorname{div}\boldsymbol{V} + (\boldsymbol{H}\nabla)V_z + \nu\Delta H_z,$$

$$\rho\Big(\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\,\nabla)\boldsymbol{V}\Big) = -\nabla(p + H_z^2/2) - \Delta A\nabla A, \qquad \rho\Big(\frac{\partial V_z}{\partial t} + (\boldsymbol{V}\,\nabla)V_z\Big) = (\boldsymbol{H}\nabla)H_z,$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho\boldsymbol{V}) = 0, \qquad \frac{\partial p}{\partial t} + (\boldsymbol{V}\nabla p) = -\gamma p \operatorname{div}\boldsymbol{V} + (\gamma - 1)\nu \boldsymbol{j}^2 + \chi\Delta T,$$

$$T = \frac{p}{\rho}, \qquad \boldsymbol{H} = (H_x, H_y) = \Big(\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}\Big), \qquad \boldsymbol{V} = (V_x, V_y).$$

The initial conditions corresponded to the steady solution of these equations $A = A_0 = (x^2 - y^2)/2$ (X-point): $H_z = H_0$, $p = \beta$, $\rho = 1$, and $\boldsymbol{V} = 0$, where H_0 and β are certain prescribed quantities.

The motion was initiated by perturbation of the z-component of the vector-potential of the magnetic field A at the boundary of the computational domain $-1 < x < 1$, $-1 < y < 1$, which corresponds to a converging cylindrical wave: $A(x,y,t) = A_0(x,y) + f(t + \ln r)$. Here $r = \sqrt{x^2 + y^2}$ (x and y belong to the boundary). The function $f(\xi)$ has the form $-E_1(\xi - \ln\sqrt{2})^2/\xi$ for $\xi > \ln\sqrt{2}$ and $f(\xi) = 0$ for $\xi < \ln\sqrt{2}$. The density ρ and plasma pressure p at the boundary were assumed equal to their undisturbed values if the plasma enters the computational domain. Otherwise, the derivatives normal to the boundary were supposed equal to zero.

The magnetic viscosity ν and the thermal conductivity χ were assumed constant.

The numerical solution of the posed problem was obtained using an explicit first-order scheme with account of the sign of velocity [1, 2].

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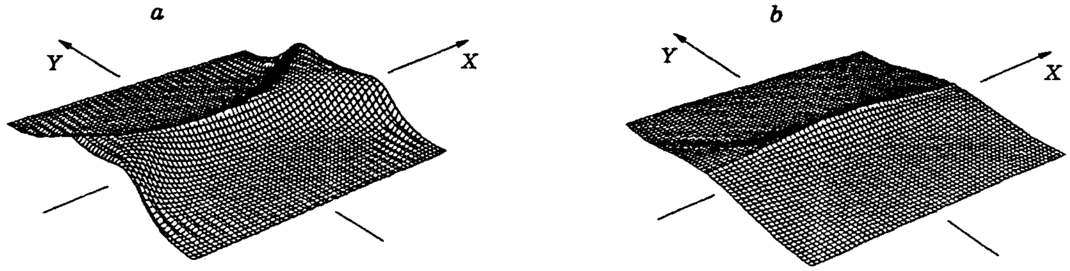

Fig. 1

The problem under study was considered in detail by Brushlinskii et al. [1] for small β and $H_0 = 0$. It was shown there that the distribution of the z-component of electric current (function ΔA) at a quasistationary stage is determined by the perturbation amplitude E_1 and inverse conductivity of the plasma ν and is almost independent of the thermal conductivity χ. According to the results of the present work, not only A, but also the total pressure $P = p + H_z^2/2 - \beta - H_0^2/2$ at large times ($t > 10$), practically depends only on E_1 and ν in a wide range of parameters $0 < \beta < 1$, $0 < H_0 < 4$, and $0 < \chi < 2\nu$. For example, for $E_1 = 0.06$ and $\nu = 0.01$, a typical distribution of P in the above-mentioned range is shown in Fig. 1a. In this case, the current layer is aligned along the X axis. The distribution of p significantly depends on the values of χ, β, and H_0. The distribution of p coincides with that shown in Fig. 1a for $H_0 = 0$ and in Fig. 1b for $H_0 = 4$.

Apparently, the reason for this behavior of P is that the poloidal velocity field arises due to reconnection of oppositely directed poloidal magnetic flows and is determined by the configuration of the poloidal magnetic field. In turn, the poloidal magnetic field is determined only by the poloidal velocity field, the perturbation of the field at the boundary E_1, and the conductivity ν. The role of other quantities is insignificant. Therefore, $\nabla(p + H_z^2/2)$ weakly depends on the parameters β, H_0, and χ. Hence, P also depends only weakly on these parameters.

Brushlinskii et al. [1] state that, for low β and $H_0 = 0$, the density at the center of the current layer reaches a minimum at $\chi \ll \nu$ and a maximum at $\chi > \nu$. It follows from our calculations that this property is observed in a wider range of the parameters β and H_0. In particular, as H_0 increases, the change in ρ significantly decreases (the transition to the limit of incompressible fluid [3]). However, the fact that the density has a minimum or a maximum in the current layer is determined only by the value of the thermal conductivity.

The above-described property of the density can be easily explained for the case of $H_z = 0$ ($H_0 = 0$) and small β. In this case, $P = p - \beta \approx p$. As mentioned above, the distribution of $P \approx p$ weakly depends on the thermal conductivity. Since the effect of thermal conductivity on temperature is quite significant, the plasma density $\rho = p/T$ also depends significantly on χ. For $H_0 \neq 0$, the discussed property of density distribution is not obvious.

2. As an example of spontaneous reconnection, we consider the development of a helical Thirring mode. The problem was solved in a cylindrical coordinate system in the region $0 \leqslant r \leqslant 1$, $0 \leqslant \varphi \leqslant 2\pi$. The presence of helical symmetry was assumed: $\partial/\partial z = -R^{-1}\partial/\partial\varphi$. In this case, it is convenient to introduce the components f_g and f_s for the vector $\boldsymbol{f}$, which are related to its cylindrical components f_z and f_φ as follows: $f_g = f_z + (r/R)f_\varphi$ and $f_s = f_\varphi - (r/R)f_z$. The equations have the form

$$\rho\Big(\frac{\partial V_g}{\partial t} + (\boldsymbol{V}\,\nabla)V_g\Big) = (\boldsymbol{H}\nabla)H_g;$$

$$\rho\Big(\frac{\partial V_r}{\partial t} + (\boldsymbol{V}\,\nabla)V_r - \frac{V_\varphi^2}{r}\Big) = -\frac{\partial p}{\partial r} + \Big(1 + \frac{r^2}{R^2}\Big)^{-1}\Big(-\frac{\partial H_g^2/2}{\partial r} + j_g\frac{\partial A_g}{\partial r}\Big); \tag{2.1}$$

$$\rho\Big(\frac{\partial V_s}{\partial t} + (\boldsymbol{V}\,\nabla)V_s + \frac{V_sV_r}{r} - 2\frac{V_rV_z}{R}\Big) = -\Big(1 + \frac{r^2}{R^2}\Big)\frac{1}{r}\frac{\partial p}{\partial\varphi} + \Big(-\frac{1}{r}\frac{\partial H_g^2/2}{\partial\varphi} + j_g\frac{1}{r}\frac{\partial A_g}{\partial\varphi}\Big); \tag{2.2}$$

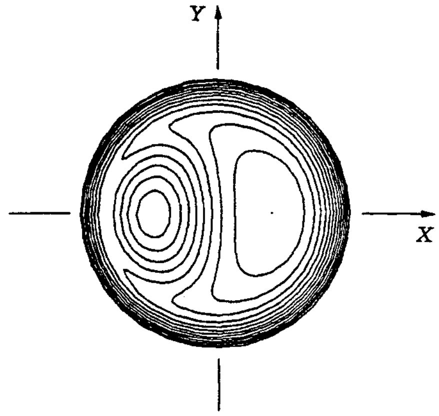

Fig. 2

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{V}) = 0; \qquad \frac{\partial A_g}{\partial t} + (\boldsymbol{V}\,\nabla)A_g = \nu\Big(\Delta A_g - 2\frac{H_z}{R}\Big);$$
$$\frac{\partial H_z}{\partial t} + \operatorname{div}(\boldsymbol{V} H_z) = (\boldsymbol{H}\nabla)V_z + \nu\Delta H_z; \tag{2.3}$$

$$H_s = -\frac{\partial A_g}{\partial r}, \quad H_r = \frac{1}{r}\frac{\partial A_g}{\partial \varphi}; \tag{2.4}$$
$$j_s = -\frac{\partial H_g}{\partial r}, \quad j_r = \frac{1}{r}\frac{\partial H_g}{\partial \varphi}, \quad j_g = -\Delta A_g + 2\frac{H_z}{R}.$$

Here A_g is the g-component of the vector-potential of the magnetic field.

Since in most tokamaks the thermal conductivity along the magnetic field is large and across the field it is very small, we used the temperature equation

$$\frac{3}{2}\rho\Big(\frac{\partial T}{\partial t} + (\boldsymbol{V}\nabla)T\Big) = -p\operatorname{div}\boldsymbol{V} + \nu \boldsymbol{j}^2 + \operatorname{div}(\chi_{\|}\boldsymbol{e}(\boldsymbol{e}\nabla)T), \qquad \boldsymbol{e} = \frac{\boldsymbol{H}}{\sqrt{H_r^2 + H_\varphi^2 + H_z^2}},$$

where $\chi_{\|}$ is the thermal conductivity along the magnetic field and $\boldsymbol{e}$ is the unit vector directed along the magnetic field.

The initial conditions are

$$\rho = 1, \quad H_z = 1, \quad V_z = 0, \quad H_r = 0, \quad H_s = \frac{r}{R}\Big(\frac{1-(1-r^2)^{q+1}}{qr^2} - 1\Big). \tag{2.5}$$

Conditions (2.5) determine the neutral layer: $H_s > 0$ near the coordinate axis and $H_s < 0$ at high values of r. The position of the neutral surface ($H_s = 0$) depends on the value of q. The initial value of plasma pressure is chosen to ensure plasma equilibrium in the magnetic field. The equilibrium was violated by a small perturbation of velocity whose form is not important.

This problem is intimately connected with saw-tooth oscillations in the tokamak [3, 4]. In this case, the parameter R is equal to the ratio of the major radius of the tokamak to the minor radius. Figures 2–4 refer to the variant $R = 4$, $q = 3$, and $\nu = 5 \cdot 10^{-5}$. An implicit scheme in the direction along φ was used for the numerical solution of this problem. The grid nodes along r were located in semi-integer points, which allowed us to avoid difficulties imposed by the use of the cylindrical coordinate system.

Figure 2 shows the force curves of the poloidal magnetic field, which coincide with isolines A_g [see (2.4)]. The evolution of perturbations in this configuration is accompanied by the appearance of an "island," reconnected force curves of the poloidal magnetic field (Fig. 2). The motion is concentrated inside some circle of radius r_*, which roughly coincides with the circle with a zero value of the total poloidal magnetic flux at the initial time: $\int_0^{r_*} H_s(r, t=0)r\,dr = 0$ [see (2.5)]. For $r > r_*$, the plasma remains almost undisturbed.

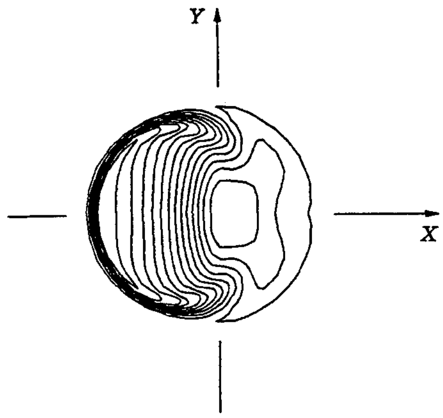

Fig. 3

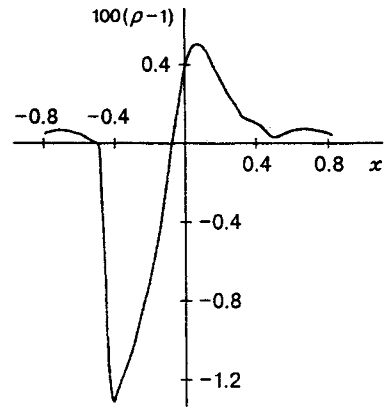

Fig. 4

The calculations revealed an interesting specific feature of the behavior of the distribution of $P = p + H_g^2/2$: except for a narrow vicinity of the current layer, the following equality is satisfied with a high accuracy:

$$(\boldsymbol{H}\nabla)P \approx 0. \tag{2.6}$$

This equality can be obtained by multiplying Eqs. (2.1) and (2.2) by H_r and H_s, respectively, and summarizing them. The poloidal velocity of plasma and, moreover, its acceleration are small (except for a narrow vicinity of the current layer); hence, taking into account that $\boldsymbol{H}\nabla A_g = 0$ and assuming $1 + r^2/R^2 \approx 1$, we obtain (2.6). Note that the value of r^2/R^2 is small even for $r = 1$ since R is large. In addition, the plasma is undisturbed for $r > r_*$; therefore, all functions in this region depend only on r and Eq. (2.6) is satisfied automatically. Thus, the neglect of the terms discussed is quite justified.

It follows from (2.6) that P is a function of A_g. The calculations show that the distribution of P is similar to the distribution of A_g even in the smallest detail, though we cannot say that P depends only on A_g and t. This is explained as follows. As mentioned above, all quantities at the periphery, where the plasma is undisturbed, depend only on r. In the remaining part of the domain, the value of dV/dt is small, except for a narrow region of the current layer. Hence, we can approximately write that $\nabla P = j_g \nabla A_g$. Except for the vicinity of the current layer, the current $j_g = -\Delta A_g + 2H_z/R$ has a constant sign and changes rather smoothly mainly in the central part of the domain, i.e., for $r < r_*$, the value of ΔA_g is approximately twice as low as $2H_z/R \sim 2/R$. Hence, we can write with a certain accuracy that $j_g \approx 2/R$ and, hence, $\nabla P \approx 2\nabla A_g/R$. At times when the island is big enough, the poloidal magnetic field (the gradients of A_g) in the center of the domain decrease. If the internal and external magnetic fluxes were equal, the poloidal magnetic field would completely disappear as a result of the reconnection. Therefore, the condition $j_g \approx 2/R$ is fulfilled with a high accuracy in the center of the domain at this stage of the process. Correspondingly, $P - 2A_g/R = \text{const}$ except for a narrow vicinity of the current layer.

The thermal conductivity has a significant effect on the temperature and pressure and levels them out along the magnetic field. The behavior of the poloidal velocity and the forces, which determine this velocity, i.e., the functions A_g and P, is almost independent of $\chi_{\|}$, as in the case of forced reconnection. This independence is obvious in the model of a large toroidal magnetic field (Kadomtsev's model) [3, 4], in which the specific form of the equation for pressure (temperature) is not important, since it is determined from the condition of incompressibility of the plasma. However, the property discussed is not connected with the presence of a large toroidal field. For example, in the above problem of propagation of a magnetic acoustic wave in the vicinity of the X-point, the process of reconnection does not depend on the thermal conductivity even if there is no toroidal field altogether.

The change in density in the process of forced reconnection is small for the most important values $R > 3$ and amounts to less than 5%. Figure 3 shows the density isolines ρ. The dependence $\rho(x)$ for $y = 0$

at the time corresponding to the magnetic configuration shown in Fig. 2 is plotted in Fig. 4. The plasma density reaches a minimum in the current layer. In contrast to the case of forced reconnection, the thermal conductivity does not lead to a distribution of ρ with a maximum in the center of the current layer. This is related to significant differences in geometry, initial pressure distribution, etc., in these two problems.

Thus, it is shown in the present paper that the total pressure behavior in the process of forced and spontaneous reconnection is mainly determined by the characteristics of the poloidal magnetic field and weakly depends on the remaining parameters of the problem. The density distribution can be significantly dependent on these parameters.

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